An Integrated Model for Survivable Network Design under Demand Uncertainty

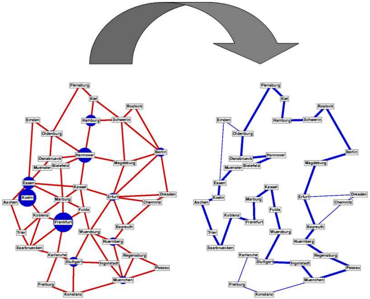
Arie M.C.A. Koster Manuel Kutschka

supported by BMBF grant 03MS616A:
ROBUKOM - Robust Communication Networks, `http://www.robukom.de`

Lehrstuhl II für Mathematik
RWTH AACHEN UNIVERSITY

11 October 2011

Network Design



given a potential network topology, demand forecast, link modules with capacities/costs

find a hardware configuration, and a routing,

such that demands are satisfied and total installation cost is minimised.

Discrete decisions:

Variations / Extensions:

- Single path routing
- Integer routing
- Survivability requirements
- Node hardware (switching capacity)
- Wavelength assignment
- Multi-layer scenarios

Integer Linear Programming formulation:

- Graph $G = (V, E)$, with nodes $i \in V$, links $e \in E$
- Commodities K of point-to-point demands $s^k \rightsquigarrow t^k$ with value d^k
- Capacity module size $C > 0$
- f_{ij}^k = fraction of demand $k \in K$ routed along arc $(i, j) \in A$

  Notation: $f_e^k := f_{ij}^k + f_{ji}^k$ for $e = ij$
- x_e = number of capacity modules to be installed on link $e \in E$

$$\min \sum_{e \in E} \kappa_e x_e$$

$$\sum_{\substack{j \in V: \\ ij \in E}} (f_{ij}^k - f_{ji}^k) = \begin{cases} 1 & i = s^k \\ -1 & i = t^k \\ 0 & \text{else} \end{cases} \quad \forall i, k$$

$$\sum_{k \in K} d^k f_e^k \leq C x_e, \quad \forall e$$

$$0 \leq f \leq 1, \; x \geq 0, \; x \in \mathbb{Z}^{|E|}$$

Goal: Protection against single-link failures

Survivable Network Design Model:

$$\min \sum_{e \in E} \kappa_e x_e$$

$$\sum_{\substack{j \in V: \\ ij \in E}} (f_{ij}^k - f_{ji}^k) = \begin{cases} z^k & i = s^k \\ -z^k & i = t^k \\ 0 & \text{else} \end{cases} \quad \forall i, k$$

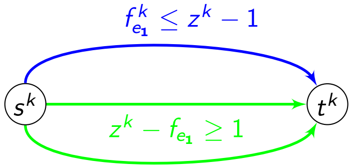
$$f_e^k \leq z^k - 1 \quad \forall e, k$$

$$\sum_{k \in K} d^k f_e^k \leq C x_e, \quad \forall e$$

$$0 \leq f \leq 1, \ z, x \geq 0, \ x \in \mathbb{Z}^{|E|}$$

Diversification of flow:

- Consider demand scaled by $z^k \geq 1$

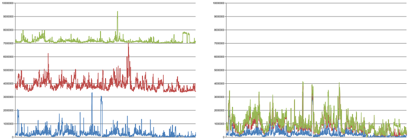


- Limit link flow s. t. 100% survives
- $z^k = 2$ with $f \in \{0, 1\}$ corresponds to 1+1 protection

Note: Given a solution (f^*, z^*, x^*), a routing template can be obtained by normalizing the flows: f_e^{k*}/z^k

Goal: Robustness against traffic fluctuations

- Traffic fluctuates heavily over time



  but only with few simultaneous peaks

- Let demand $d^k \in [0, \bar{d}^k + \hat{d}^k]$ with nominal demand $\bar{d}^k$ and deviation $\hat{d}^k$
- assume at most Γ peaks at same time

Γ-Robust Network Design Model:

$$\min \sum_{e \in E} \kappa_e x_e$$

$$\sum_{\substack{j \in V: \\ ij \in E}} (f_{ij}^k - f_{ji}^k) = \begin{cases} 1 & i = s^k \\ -1 & i = t^k \\ 0 & \text{else} \end{cases} \quad \forall i, k$$

$$\sum_{k \in K} \bar{d}^k f_e^k + \max_{\substack{Q \subseteq K \\ |Q| \leq \Gamma}} \sum_{k \in Q} \hat{d}^k f_e^k \leq C x_e \quad \forall e$$

$$0 \leq f \leq 1, \quad x \geq 0, \quad x \in \mathbb{Z}^{|E|}$$

Note: max-term can be linearized by exponential many constraints

Goal: Determine model for Survivable Γ-Robust Network Design!

Survivable Network Design Model:

$$\min \sum_{e \in E} \kappa_e x_e$$

$$\sum_{\substack{j \in V: \\ ij \in E}} (f_{ij}^k - f_{ji}^k) = \begin{cases} z^k & i = s^k \\ -z^k & i = t^k \\ 0 & \text{else} \end{cases} \quad \forall i, k$$

$$f_e^k \leq z^k - 1 \qquad \forall e, k$$

$$\sum_{k \in K} d^k f_e^k \leq C x_e, \qquad \forall e$$

$$0 \leq f \leq 1, \; z, x \geq 0, \; x \in \mathbb{Z}^{|E|}$$

Γ-Robust Network Design Model:

$$\min \sum_{e \in E} \kappa_e x_e$$

$$\sum_{\substack{j \in V: \\ ij \in E}} (f_{ij}^k - f_{ji}^k) = \begin{cases} 1 & i = s^k \\ -1 & i = t^k \\ 0 & \text{else} \end{cases} \quad \forall i, k$$

$$\sum_{k \in K} \bar{d}^k f_e^k + \max_{\substack{Q \subseteq K \\ |Q| \leq \Gamma}} \sum_{k \in Q} \hat{d}^k f_e^k \leq C x_e \quad \forall e$$

$$0 \leq f \leq 1, \; x \geq 0, \; x \in \mathbb{Z}^{|E|}$$

Goal: Determine model for Survivable Γ-Robust Network Design!

1+1 protected Network Design Model:

$$\min \sum_{e \in E} \kappa_e x_e$$

$$\sum_{\substack{j \in V: \\ ij \in E}} (f_{ij}^k - f_{ji}^k) = \begin{cases} 2 & i = s^k \\ -2 & i = t^k \\ 0 & \text{else} \end{cases} \quad \forall i, k$$

$$f_e^k \leq 2 - 1 \quad \forall e, k$$

$$\sum_{k \in K} d^k f_e^k \leq C x_e, \quad \forall e$$

$$0 \leq f \leq 1, \quad x \geq 0, \; x \in \mathbb{Z}^{|E|}$$

Γ-Robust Network Design Model:

$$\min \sum_{e \in E} \kappa_e x_e$$

$$\sum_{\substack{j \in V: \\ ij \in E}} (f_{ij}^k - f_{ji}^k) = \begin{cases} 1 & i = s^k \\ -1 & i = t^k \\ 0 & \text{else} \end{cases} \quad \forall i, k$$

$$\sum_{k \in K} \bar{d}^k f_e^k + \max_{\substack{Q \subseteq K \\ |Q| \leq \Gamma}} \sum_{k \in Q} \hat{d}^k f_e^k \leq C x_e \quad \forall e$$

$$0 \leq f \leq 1, \; x \geq 0, \; x \in \mathbb{Z}^{|E|}$$

Goal: Determine model for Survivable Γ-Robust Network Design!

1+1 protected Network Design Model:

$$\min \sum_{e \in E} \kappa_e x_e$$

$$\sum_{\substack{j \in V: \\ ij \in E}} (f_{ij}^k - f_{ji}^k) = \begin{cases} 2 & i = s^k \\ -2 & i = t^k \\ 0 & \text{else} \end{cases} \quad \forall i, k$$

$$\sum_{k \in K} d^k f_e^k \leq C x_e, \quad \forall e$$

$$0 \leq f \leq 1, \quad x \geq 0, \ x \in \mathbb{Z}^{|E|}$$

Γ-Robust Network Design Model:

$$\min \sum_{e \in E} \kappa_e x_e$$

$$\sum_{\substack{j \in V: \\ ij \in E}} (f_{ij}^k - f_{ji}^k) = \begin{cases} 1 & i = s^k \\ -1 & i = t^k \\ 0 & \text{else} \end{cases} \quad \forall i, k$$

$$\sum_{k \in K} \bar{d}^k f_e^k + \max_{\substack{Q \subseteq K \\ |Q| \leq \Gamma}} \sum_{k \in Q} \hat{d}^k f_e^k \leq C x_e \quad \forall e$$

$$0 \leq f \leq 1, \ x \geq 0, \ x \in \mathbb{Z}^{|E|}$$

Γ-Robust Network Design Model with 1+1 protection:

$$\min \quad \sum_{e \in E} \kappa_e x_e$$

$$\sum_{\substack{j \in V: \\ ij \in E}} (f_{ij}^k - f_{ji}^k) \quad = \begin{cases} 2 & i = s^k \\ 2 & i = t^k \;\; \forall i, k \\ 0 & \text{else} \end{cases}$$

$$\sum_{k \in K} \bar{d}^k f_e^k + \max_{\substack{Q \subseteq K \\ |Q| \leq \Gamma}} \sum_{k \in Q} \hat{d}^k f_e^k \leq C x_e \forall e$$

$$0 \leq f \leq 1, \;\; x \geq 0, \;\; x \in \mathbb{Z}^{|E|}$$

Notes:

- max-term can be linearized by exponential many constraints
- a compact linear reformulation can also be obtained by linear duality

Observations:

- additional dedicated capacity needed to protect against peaks
- many (simultaneous) peaks are rare
- additional dedicated capacity needed for survivability
- single-link failures are rare

Observations:

- additional dedicated capacity needed to protect against peaks
- many (simultaneous) peaks are rare
- additional dedicated capacity needed for survivability
- single-link failures are rare

Goal:

- Use additional capacity for both Γ-Robustness and Survivability
- Guarantee survivability only for nominal demands, not for peaks

$$\text{during normal operation:} \quad \sum_{k\in K} \bar{d}^k \frac{f_e^k}{z^k} + \max_{\substack{Q\subseteq K \\ |Q|\leq \Gamma}} \sum_{k\in Q} \hat{d}^k \frac{f_e^k}{z^k} \leq C x_e \quad \forall e$$

$$\text{during failure:} \quad \sum_{k\in K} \bar{d}^k f_e^k \leq C x_e \quad \forall e$$

Survivable Network Design Model:

$$\min \sum_{e \in E} \kappa_e x_e$$

$$\sum_{\substack{j \in V: \\ ij \in E}} (f_{ij}^k - f_{ji}^k) = \begin{cases} z^k & i = s^k \\ -z^k & i = t^k \\ 0 & \text{else} \end{cases} \quad \forall i, k$$

$$f_e^k \le z^k - 1 \quad \forall e, k$$

$$\sum_{k \in K} d^k f_e^k \le C x_e, \quad \forall e$$

$$0 \le f \le 1, \; z, x \ge 0, \; x \in \mathbb{Z}^{|E|}$$

Γ-Robust Network Design Model:

$$\min \sum_{e \in E} \kappa_e x_e$$

$$\sum_{\substack{j \in V: \\ ij \in E}} (f_{ij}^k - f_{ji}^k) = \begin{cases} 1 & i = s^k \\ -1 & i = t^k \\ 0 & \text{else} \end{cases} \quad \forall i, k$$

$$\sum_{k \in K} \bar{d}^k f_e^k + \max_{\substack{Q \subseteq K \\ |Q| \le \Gamma}} \sum_{k \in Q} \hat{d}^k f_e^k \le C x_e \quad \forall e$$

$$0 \le f \le 1, \; x \ge 0, \; x \in \mathbb{Z}^{|E|}$$

Survivable Γ-Robust Network Design Model:

$$\min \sum_{e \in E} \kappa_e x_e$$

$$\sum_{\substack{j \in V: \\ ij \in E}} (f_{ij}^k - f_{ji}^k) = \begin{cases} z^k & i = s^k \\ -z^k & i = t^k \\ 0 & \text{else} \end{cases} \quad \forall i, k$$

$$f_e^k \leq z^k - 1 \quad \forall e, k$$

$$\sum_{k \in K} \bar{d}^k f_e^k \leq C x_e, \quad \forall e$$

$$\sum_{k \in K} \bar{d}^k \frac{f_e^k}{z^k} + \max_{\substack{Q \subseteq K \\ |Q| \leq \Gamma}} \sum_{k \in Q} \hat{d}^k \frac{f_e^k}{z^k} \leq C x_e \quad \forall e$$

$$0 \leq f \leq 1, \quad z, x \geq 0, \quad x \in \mathbb{Z}^{|E|}$$

Survivable Γ-Robust Network Design Model:

$$\min \sum_{e\in E} \kappa_e x_e$$

$$\sum_{\substack{j\in V:\\ ij\in E}} (f_{ij}^k - f_{ji}^k) = \begin{cases} z^k & i = s^k \\ -z^k & i = t^k \\ 0 & \text{else} \end{cases} \quad \forall i,k$$

$$f_e^k \leq z^k - 1 \quad \forall e,k$$

$$\sum_{k\in K} \bar{d}^k f_e^k \leq C x_e, \quad \forall e$$

$$\sum_{k\in K} \bar{d}^k \frac{f_e^k}{z^k} + \max_{\substack{Q\subseteq K\\ |Q|\leq\Gamma}} \sum_{k\in Q} \hat{d}^k \frac{f_e^k}{z^k} \leq C x_e \quad \forall e$$

$$0 \leq f \leq 1, \; z, x \geq 0, \; x \in \mathbb{Z}^{|E|}$$

Note: max-term and $\frac{f_e^k}{z^k}$ are nonlinear!

Goal: Linearize

$$\sum_{k \in K} \bar{d}^k \frac{f_e^k}{z^k} + \max_{\substack{Q \subseteq K \\ |Q| \leq \Gamma}} \sum_{k \in Q} \hat{d}^k \frac{f_e^k}{z^k} \leq C x_e \quad \forall e \qquad (*)$$

Relaxation of $\frac{f_e^k}{z^k}$:

- Let $\lambda^k :=$ # disjoint paths from s^k to t^k, then $z^k \geq \frac{\lambda^k}{\lambda^k - 1}$ must hold
- Thus, $(*)$ can be relaxed to

$$\sum_{k \in K} \bar{d}^k \left(1 - \frac{1}{\lambda^k}\right) f_e^k + \max_{\substack{Q \subseteq K \\ |Q| \leq \Gamma}} \sum_{k \in Q} \hat{d}^k \left(1 - \frac{1}{\lambda^k}\right) f_e^k \leq C x_e \quad \forall e$$

Linearization of max-term:

- Compact linear reformulation using LP duality

Relaxed reformulation of Survivable Γ-Robust Network Design Model:

$$\min \sum_{e \in E} \kappa_e x_e$$

$$\sum_{\substack{j \in V: \\ ij \in E}} (f_{ij}^k - f_{ji}^k) = \begin{cases} z^k & i = s^k \\ -z^k & i = t^k \\ 0 & \text{else} \end{cases} \quad \forall i, k$$

$$f_e^k \leq z^k - 1 \quad \forall e, k$$

$$\sum_{k \in K} \bar{d}^k f_e^k \leq C x_e, \quad \forall e$$

$$\sum_{k \in K} \bar{d}^k \left(1 - \frac{1}{\lambda^k}\right) f_e^k + \Gamma \pi_e + \sum_{k \in K} p_e^k \leq C x_e \quad \forall e$$

$$\pi_e + p_e^k \geq \hat{d}^k \left(1 - \frac{1}{\lambda^k}\right) f_e^k \quad \forall e, k$$

$$0 \leq f \leq 1, \quad \pi, p, z, x \geq 0, \quad x \in \mathbb{Z}^{|E|}$$

Survivable Γ-Robust Network Design Model:

$$\min \quad \sum_{e \in E} \kappa_e x_e$$

$$\sum_{\substack{j \in V: \\ ij \in E}} (f_{ij}^k - f_{ji}^k) = \begin{cases} z^k & i = s^k \\ -z^k & i = t^k \\ 0 & \text{else} \end{cases} \quad \forall i, k$$

$$f_e^k \leq z^k - 1 \quad \forall e, k$$

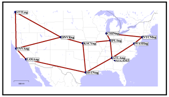
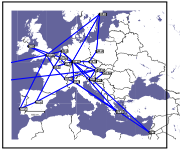
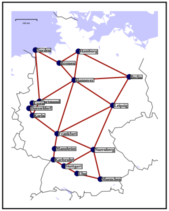
$$\sum_{k \in K} \bar{d}^k f_e^k \leq C x_e, \quad \forall e$$

$$\sum_{k \in K} \bar{d}^k \left(1 - \frac{1}{\lambda^k}\right) f_e^k + \Gamma \pi_e + \sum_{k \in K} p_e^k \leq C x_e \quad \forall e$$

$$\pi_e + p_e^k \geq \hat{d}^k \left(1 - \frac{1}{\lambda^k}\right) f_e^k \quad \forall e, k$$

$$0 \leq f \leq 1, \quad \pi, p, z, x \geq 0, \quad x \in \mathbb{Z}^{|E|}$$

Problem instances:

Network	Abilene	GÉANT	Germany17
Topology			
# nodes	12	22	17
# links	15	36	26
# demands	66	231	136
Instances	abilene1	geant1	ger17
	abilene2	geant2	

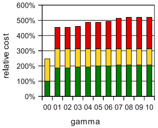
Environment: C++, IBM ILOG CPLEX 12.1, 2.93 GHz CPU, 12 GB RAM, 12h timelimit per instance

Computational Results: Costs

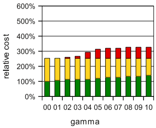
- ■ Γ-Robust Network Design with 1+1 protection
- ■ Survivable Γ-Robust Network Design
- ■ Γ-Robust Network Design

Cost of survivability

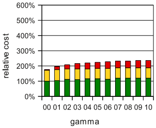
abilene1



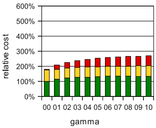
abilene2



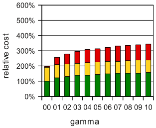
geant1



geant2



ger17



Cost savings for $\Gamma > 0$ by using ■ instead of ■

[31%,40%]	[0%,22%]	[10%,20%]	[14%,24%]	[19%,30%]

Cost savings $\geq 15\%$ by using ■ instead of ■, if

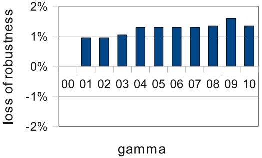
$\Gamma \geq 1$	$\Gamma \geq 5$	$\Gamma \geq 3$	$\Gamma \geq 2$	$\Gamma \geq 1$

Computational Results: Robustness

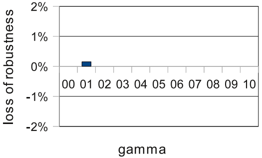
Realized Robustness: percentage of traffic matrices that can be accomodated using routing and link dimensioning

Loss of realized robustness by using ■ (yellow) instead of ■ (red)

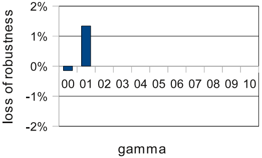
abilene1



geant1



geant2



Notes:

- no loss of robustness for abilene2 and ger17
- highest observed loss has been 1.6% (abilene1, $\Gamma = 9$)

Conclusions:

- Model for Γ-Robust Network Design with 1+1 protection
- Use additional link capacity for both robustness and protection
- Integrated model for Survivable Γ-Robust Network Design
- Cost savings up to 40%
- Insignificant loss of robustness (less than 1.6%)

Further information: `http://www.robukom.de`

An Integrated Model for Survivable Network Design under Demand Uncertainty

Arie M.C.A. Koster   Manuel Kutschka

supported by BMBF grant 03MS616A:
ROBUKOM - Robust Communication Networks, `http://www.robukom.de`

Lehrstuhl II für Mathematik

RWTH AACHEN UNIVERSITY

11 October 2011